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International Conference on Contents, Computing & Communication (ICCCC-2022)

26th and 27th February,2022

Behaviour Of Quasi Static Thermal Stresses On Account Of Internal Moving Heat Generation In Solid Sphere

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Abstract

This work emphasizes on the effect of heat generation on quasi static thermal stresses in solid sphere. This is the construction of wide mathematical treatment through a one-dimensional steady state temperature distribution and its corresponding stresses on account of thermal tension in the shape of solid sphere. It has been exposed to different types of heat sources developed. This paper elaborates on study of the effect of varying heat generation on displacement and thermal stresses. The integral transform technique has been applied to solve heat conduction equation by assuming the convective thermal boundary condition and arbitrary initial and surrounding temperature. The outcome of this analysis is represented numerically and graphically.

Keywords: Thermal stresses, integral transform, heat sources.

1. Introduction

A thermal stress on account of heat generation through solids of various dimensions studied and analyzed so far. Extensive research has been done in variation of thermal stresses with respect to changes in internal heat produced by internal heat sources when travels through the material. An engineering and industrial field remained point of attention in its applications. Thermal stresses in solid spheres have been studied by many authors. The survey of literature meets the entire study of work performed on thermal stresses, temperature distribution etc since long period of century. Boley, Wiener [1], Parkus [2], Noda [3], Nowacki [4], Carslaw and Jaeger [5] provided the plenty of analytical information. Cheung et al [7] evaluated the stresses in sphere by using local heating. The tensile stress concentrated limited to the region of heating in the interior of the solid. The stresses in magnitude corresponding to uniform heating exceed the stresses by nonuniform heating. Takeuti and Tanigawa [8] developed the model on spherical region of hollow sphere. This development is constructed on the basis of the three-dimensional field equations of motion. The thermoelastic displacement potential has been introduced in this paper. All the stress components are described in terms of this displacement potential with the aid of a harmonic function. Obata and Noda [9] attempted to find the effect of inside radius size and variation of temperature on thermal stresses in FGM hollow sphere. It underlined the the study of effect of composition material on thermal stresses. Ootao and Tanigawa [10] used spherical coordinates for development of theoretical model of treatment to thermal stress problem. It has been calculated by considering the rotating heat sources inside hollow sphere. The relations of stress components in terms of thermoelastic displacement potential and a harmonic function have been depicted. Ahire and Hamoud [11] treated rectangular plate of FGM and analysed the temperature distribution with corresponding thermal stresses. The dependency of thermal stresses on variation of temperature depicted and studied graphically. The different composition of material was considered and compared accordingly. Lutz and Zimmerman [12] presented a problem of uniform heating of sphere and studied linear variation of elastic moduli and thermal expansion coefficient with radius. Frobenius series method applied for finding the exact form of displacement and stresses. P. Rani et.al.[13] found analytical solution of radially varying properties. Thermal stress distribution studied on account of the different values of powers of module of elasticity and power law index of heat generation. Bhave et.al [14] dealt with three dimensional axisymmetric problem by providing heat to the surface of clamped sphere. Heat transfer and stresses were studied and analysed by Legendre Transform under steady temperature field. Kobayashi and Sugano [15] treated hemisphere with unaxisymmetric heating at inner and outer hemispherical surfaces. The

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stresses and analytical solution obtained by using Fourier cosine transform technique and Legendre transform.

2. Formulation of the problem:

Let a solid sphere defined in a region $a \leq r \leq a$. A sphere is assumed to be placed in arbitrary temperature initially.
The transient temperature distribution is governed by the equation:

$$\frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r} + \frac{g(r,t)}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \quad \text{in} \quad 0 \leq r < a, \quad t > 0 \tag{1}$$

Boundary and initial condition:

$$\left[\frac{\partial T}{\partial r}\right]_{r=a} = f_1(t) \quad t > 0 \tag{2}$$

$$\left[\frac{\partial T}{\partial r}\right]_{r=b} = f_2(t) \quad t > 0 \tag{3}$$

Where k is thermal conductivity, h is heat transfer and α is thermal diffusivity of the material.
The temperature is symmetric with respect to center of sphere, a function of r that is the radial distance. Spherically symmetric problem, in which shearing stresses and strains vanish and strain and strain components in spherical coordinate $\theta \; and \; \emptyset$ are identical.

$$\sigma_{\theta\theta} = \sigma_{\emptyset\emptyset}\,, \epsilon_{\theta\theta} = \epsilon_{\emptyset\emptyset} \tag{4}$$

$$\sigma_{r\theta} = \sigma_{\theta\emptyset} = \sigma_{\emptyset r} = 0 \tag{5}$$

$$\epsilon_{r\theta} = \epsilon_{\theta\emptyset} = \epsilon_{\emptyset r} = 0 \tag{6}$$

The equilibrium equation without body force in spherical coordinates reduced to

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r}[2\sigma_{rr} - \sigma_{\theta\theta} - \sigma_{\emptyset\emptyset}] = 0 \tag{7}$$

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{2}{r}[2\sigma_{rr} - \sigma_{\theta\theta}] = 0 \tag{8}$$

Stress strain relation:

$$\sigma_{rr} = \lambda_e + 2\mu\epsilon_{rr} - (3\lambda + 2\mu)a_t T \tag{9}$$

$$\sigma_{\theta\theta} = \sigma_{\emptyset\emptyset} = \lambda e + 2\mu\epsilon_{\theta\theta} - (3\lambda + 2\mu)a_t T \tag{10}$$

Where strain dilation $e = \epsilon_{rr} + \epsilon_{\theta\theta} + \epsilon_{\emptyset\emptyset} = \epsilon_{rr} + 2\epsilon_{\theta\theta}$ (11)

Where $\sigma_{rr}, \sigma_{\theta\theta} \; and \; \sigma_{\emptyset\emptyset}$ are the stresses in the radial and tangential direction and $\epsilon_{rr} \; and \; \epsilon_{\theta\theta}$ are strains in radial and tangential direction, e is the strain dilation.

λ and μ are the Lame constants related to the modulus of elasticity E and the Poisson's ratio ν as

$$\boldsymbol{\lambda} = \frac{\nu E}{(1+\nu)(1-2\nu)} \;, \boldsymbol{\mu} = \frac{\boldsymbol{E}}{2(1+\nu)} \tag{12}$$

The strain component in terms of radial displacement $u_r = u$ is

$$\epsilon_{rr} = \frac{du}{dr}, \quad \epsilon_{\theta\theta} = \epsilon_{\emptyset\emptyset} = \frac{u}{r} \tag{13}$$

The boundary condition on traction free surface is

$$\sigma_{rr} = 0 \quad at\; r = a \; and \; r = b \tag{14}$$

$$\sigma_{rr} = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\frac{du}{dr} + 2\nu\frac{u}{r} - (1+\nu)a_t T\right] \tag{15}$$

$$\sigma_{\theta\theta} = \frac{E}{(1+\nu)(1-2\nu)}\left[\nu\frac{du}{dr} + \frac{u}{r} - (1+\nu)a_t T\right] \tag{16}$$

Mathematical model is constructed by using above equations (1) to (16).
Analysis Solutions:
On applying integral transform and inverse formula to the problem, one obtains, the expression for the temperature function of a non-homogeneous boundary problem of heat conduction in sphere as,

$$T = \frac{2}{a}\sum \frac{B}{\alpha Q}(1 - e^{-\alpha Q t})\cos\left(\frac{m\pi r}{a}\right) + r^2 t e^{-\alpha Q t}\cos\left(\frac{m\pi r}{a}\right) \tag{17}$$

$$\epsilon_{rr} = \frac{-g_0 r 2t}{k(2\alpha - r)^2} \tag{18}$$

$$\epsilon_{\theta\theta} = \frac{-g_0 \alpha t}{k(2\alpha - r)} \tag{19}$$

Copper Sphere:

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$$T = 2\sum \frac{B}{\alpha Q}(1 - e^{-\alpha Q t})\cos\left(\frac{m\pi r}{a}\right) + r^2 t e^{-\alpha Q t}\cos\left(\frac{m\pi r}{a}\right) \quad (20)$$

where

$$B = 800.079 - \frac{6080}{r} \;;\quad Q = \pi^2 - \frac{\pi}{r} \quad ;\alpha = 16$$

$$\sigma_{rr} = 2.8 \times 10^8 \left[\frac{-0.05267t}{(32-r)^2} + \frac{-10.88t}{(12832-401r)} - 21.44T\right] \quad (21)$$

$$\sigma_{\theta\theta} = 2.8 \times 10^8 \left[\frac{-0.027t}{(32-r)^2} + \frac{-16t}{(12832-401r)} - 21.44T\right] \quad (22)$$

Aluminum Sphere:

$$T = 2\sum \frac{B}{\alpha Q}(1 - e^{-\alpha Q t})\cos\left(\frac{m\pi r}{a}\right) + r^2 t e^{-\alpha Q t}\cos\left(\frac{m\pi r}{a}\right) \quad (23)$$

where

$$B = 1050.1779 - \frac{7980}{r} \;;\quad Q = \pi^2 - \frac{\pi}{r} \quad ;\alpha = 21$$

$$\sigma_{rr} = 170.3703 \times 10^6 \left[\frac{-0.1156t}{(42-r)^2} + \frac{-0.06228t}{(42-r)} - 28.35T\right] \quad (24)$$

$$\sigma_{\theta\theta} = 170.3703 \times 10^6 \left[\frac{-0.06228t}{(42-r)^2} + \frac{-0.08898t}{(42-r)} - 28.35T\right] \quad (25)$$

Cobalt Sphere:

$$T = 2\sum \frac{B}{\alpha Q}(1 - e^{-\alpha Q t})\cos\left(\frac{m\pi r}{a}\right) + r^2 t e^{-\alpha Q t}\cos\left(\frac{m\pi r}{a}\right) \quad (26)$$

where

$$B = 600.2307 - \frac{4560}{r} \;;\quad Q = \pi^2 - \frac{\pi}{r} \quad ;\alpha = 12$$

$$\sigma_{rr} = 3.9975 \times 10^6 \left[\frac{-0.1592t}{(24-r)^2} + \frac{0.0715t}{(24-r)} - 15.72T\right] \quad (27)$$

$$\sigma_{\theta\theta} = 3.9975 \times 10^6 \left[\frac{-0.0715t}{(24-r)^2} + \frac{-0.11538t}{(24-r)} - 15.72T\right] \quad (28)$$

Chromium Sphere:

$$T = 2\sum \frac{B}{\alpha Q}(1 - e^{-\alpha Q t})\cos\left(\frac{m\pi r}{a}\right) + r^2 t e^{-\alpha Q t}\cos\left(\frac{m\pi r}{a}\right) \quad (29)$$

where

$$B = 300.1266 - \frac{2280}{r} \;;\quad Q = \pi^2 - \frac{\pi}{r} \quad ;\alpha = 6$$

$$\sigma_{rr} = 340.2778 \times 10^6 \left[\frac{-0.1012t}{(12-r)^2} + \frac{-0.0253t}{(12-r)} - 7.2T\right] \quad (30)$$

$$\sigma_{\theta\theta} = 340.2778 \times 10^6 \left[\frac{-0.0253t}{(12-r)^2} + \frac{-0.06329t}{(12-r)} - 7.2T\right] \quad (31)$$

3. Result and Discussion

Graphical Analysis:

Aluminum:

Aluminum is sensitive to change of temperature. As depicted in fig. 1 and fig.2 The behavior of thermal stresses along the axis and radius are found to be same in nature. But their capacity of handling the stress is different in both directions. The maximum limiting value found more along the axis as compare to the radial.

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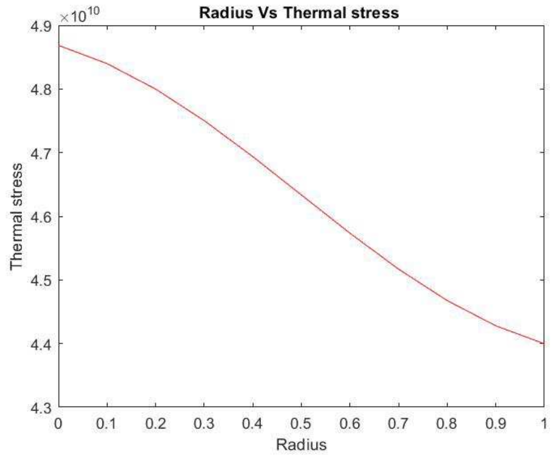


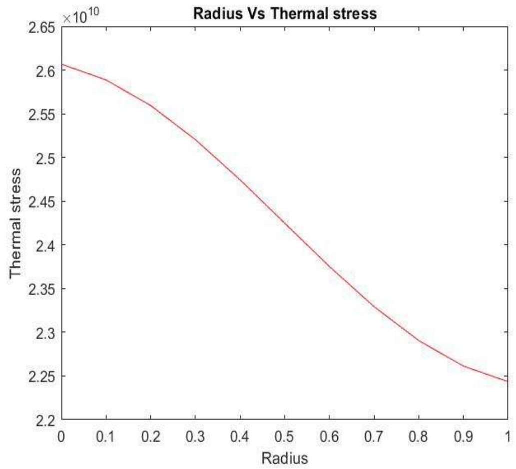
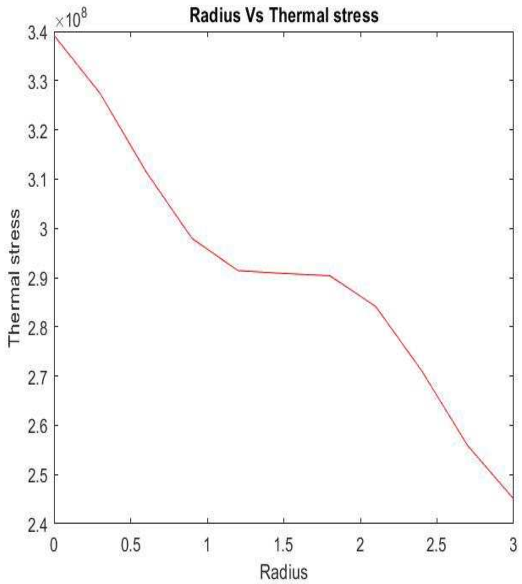


Fig.1)Radius Vs Thermal stress along axis *Fig.2) Radius Vs Thermal stress along radius*

Cobalt:



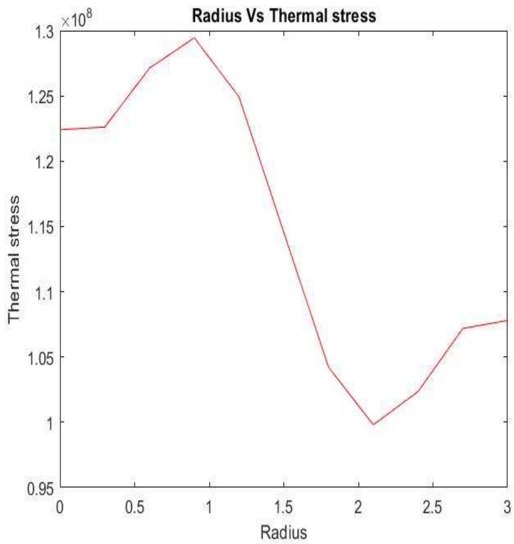


Fig.3)Radius Vs Thermal stress along axis *Fig.4) Radius Vs Thermal stress along radius*

Cobalt is known for increase in temperature handling capacity.As observed in graph thermal stresses reduced

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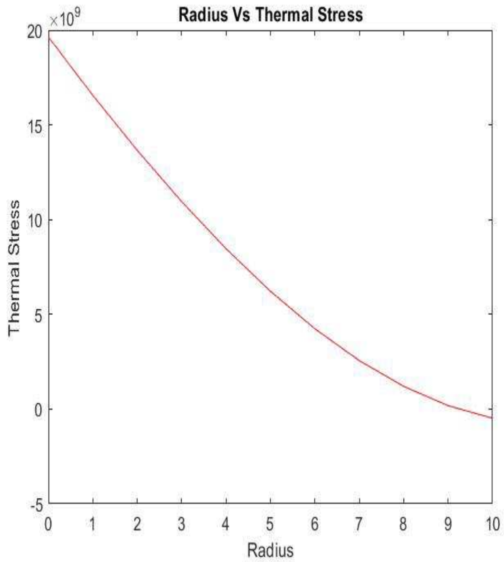
along the axis and radial behaviour is not uniformly changing.As found it is quit oscilating from maximum to minimum but remain active to the change in temperature and corresponding thermal stresses.

Chromium:

In this the same result is depicted. The thermal stress starts falling from its maximum to minimum along the radius. Along the axis uniform fall from higher to lower thermal stress is observed. The directional balance of change in thermal stresses is observed.

As observed from the graph copper is not showing uniform relation in handling the thermal stresses along the axis. The journey of thermal stresses along the radius is not so gradual. It falls and rise alternately indicates the nonuniform effect. The overall response of copper predicts the variation in thermals stresses in different direction along the axis or radius.



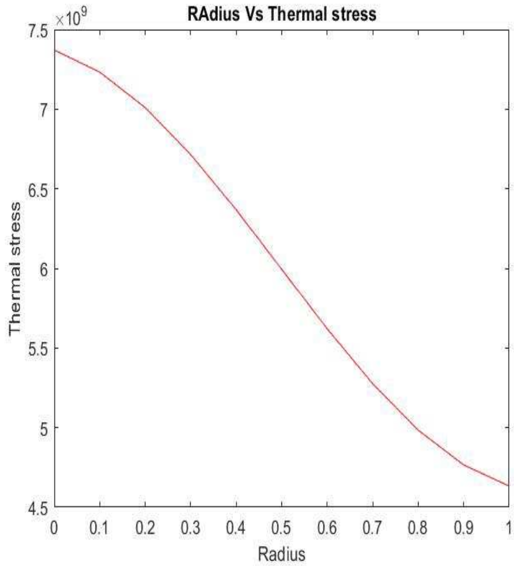
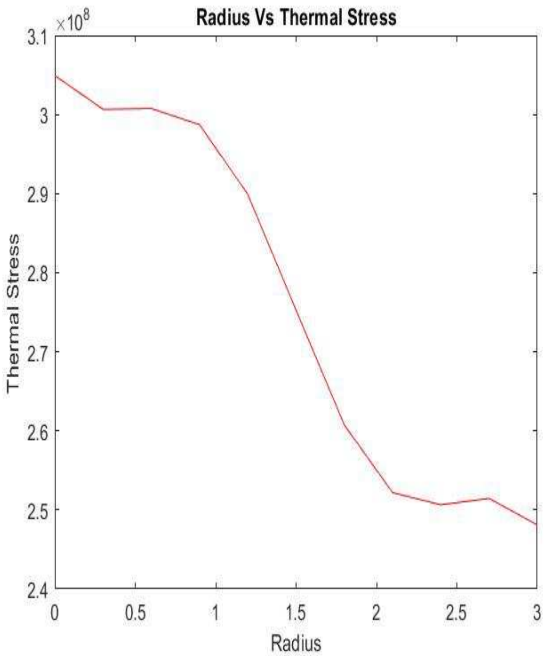


Fig.5) Radius Vs Thermal Stress along axis

Fig.6) Radius Vs Thermal Stress along radius

Copper:



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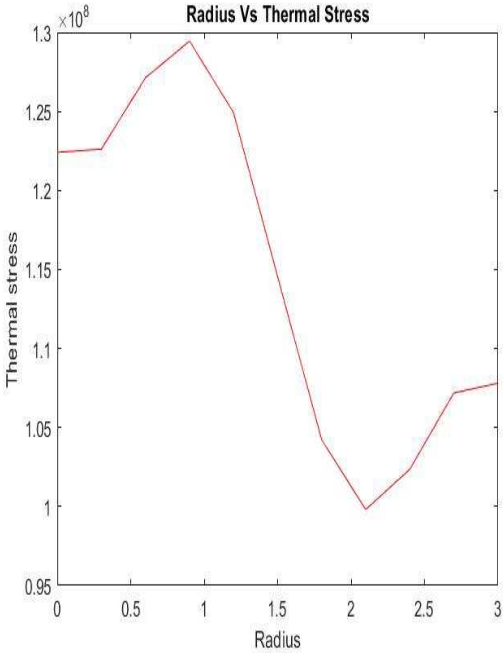

Fig.7) Radius Vs Thermal Stress along axis *Fig.8) Radius VsThermal Stress along radius*

4. Conclusion:

In this article analysis is done by considering the shape of sphere. For this four materials were treated mathematically for determination of temperature and thermal stresses using integral transform technique. The second kind boundary conditions have been used for solving the heat equation for internal heat generation. The axial and radial thermal stresses studied numerically and graphically. When these four materials compared it is found that the stress handling capacity of copper and aluminum is more as compare to chromium and cobalt.

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